U/2/7/48 1

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On the Notation of Relations

The traditional arrow notation seems to represent duplex chains perfectly adequately. It is true that it is not logically exhaustive:

1. $$\begin{array}{lll} a \leftarrow b = (a + ab) & = (a\underline{b} + ab + \underline{ab}) & (\underline{\text{note}}\text{: } \underline{a} = \text{'not-a'}) \\ a \rightarrow b = (ab + b) & = (ab + \underline{a}b + \underline{ab}) & \\ a \leftrightarrow b = (ab) & = (ab + \underline{ab}) & \\ a - b = (a + ab + b) & = (a\underline{b} + ab + \underline{a}b) & \\ \qquad\quad (a + b) & = (a\underline{b} + \underline{a}b) & \\ \qquad\quad (a) & = (a\underline{b} + \underline{ab}) & \\ \qquad\quad (b) & = (\underline{a}b + \underline{ab}) & \\ \qquad\quad (\ ) & \quad (\underline{ab}) & \end{array}$$

but it can be maintained with some semblance of reason that in the last four cases there is no relation between a and b.

It has been supposed up till now that the same notation would work equally well for chains of higher complexity, since, by various arrangements of brackets, each relation could always be represented as having two and only two terminals. It does work in some cases, for instance in the analysis of the Spanish syllable [tʃe], where the class needed is (tʃe + te + e):

2. $(a \leftarrow b) \rightarrow c = (abc + ac + c)$

However, as indicated in U/20/6/47 1-2, there are 52 possible classes to which no arrangement of arrows and brackets corresponds, and it would now appear that some, if not all, of these unrepresented classes actually occur. Here is an example:

The nexis "when he came I went" can be analysed as follows: 1. (when he came)(I went); 2. ((when)(he came))(I went), i.e. (ab)c. The class needed is (abc + b + bc + c), since the possible nexias are "when he came I went", "he came", "he came, I went", and "I went", but no arrangement of arrows will produce this class. It can be done by the composite arrangement: ×

3. $((a \leftrightarrow b) \rightarrow c) + (b - c) = (abc + c) + (b + bc + c) = (abc + b + bc + c)$

since 2c = c. But this is unsatisfactory because it does not indicate the true orientation of the relation between a and b nor the ambivalence of the relation between (ab + b) and c:

The relation between a and b is obviously

4. $a \rightarrow b = (ab + b)$

since the two possible nexus are "when he came" and "he came". The relation between (a → b) and c is ambivalent and selective since, when the relation has the orientation (a → b) → c the value of (a → b) must be ab, and when the orientation is (a → b) – c the value of (a → b) must be b. In other words, c enters into relation not with one orientation but with a class of orientations, via (→ + –)c. This could be expressed as follows:

$$\begin{pmatrix} (a \leftrightarrow b) \rightarrow \\ \quad (b) - \end{pmatrix} c$$

which still does not solve the problem of adequate representation of the relation (a → b).

× $(b \rightarrow\leftarrow c + b \leftarrow\rightarrow c) \leftarrow\leftarrow a = (abc + bc + b + c)$ 13/8/48